

A possible scenario of the Pioneer anomaly in the framework of Finsler geometry

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Abstract

The weak field approximation of geodesics in Randers-Finsler space is investigated. We show that a Finsler structure of Randers space corresponds to the constant and sunward anomalous acceleration demonstrated by the Pioneer 10 and 11 data. The additional term in the geodesic equation acts as “electric force”, which provides the anomalous acceleration.

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Newton's theory of gravitation was proposed almost three hundred and fifty years ago. Einstein's general relativity reveals the intrinsic geometric property of gravity. Newton's theory of gravitation is the main guideline of the celestial mechanics, especially for the solar system. General relativity provides small corrections. It is well-known that the Newton and Einstein's theory of gravitation still faces problems. One of them is that the flat rotation curves of spiral galaxies violate the prediction of Newton and Einstein's gravity. Another is related with recent astronomical observations[1]. Our universe is acceleratedly expanding. This result can not be obtained directly from Einstein's gravity and his cosmological principle. In fact, new puzzle has also arisen in the solar system. That is the Pioneer anomaly. The Pioneer spacecrafts are excellent tools for dynamical astronomy studies in the solar system. The radio metric data from the Pioneer 10/11 spacecraft indicate the presence of a small, anomalous, Doppler frequency drift over the range of 20–70 astronomical units[2]. The drift is blue-shift, uniformly changing with a rate of $6 \times 10^{-9} Hz/s$ [3]. It has revealed an anomalous constant sunward acceleration, $a_P = (8.74 \pm 1.33) \times 10^{-10} m/s^2$.

The Pioneer data have been studied in three different navigational computer programs. Namely: the JPL's Orbit Determination Program (ODP), the Aerospace Corporation's CHASPM code extended for deep space navigation[2], and a code written in the Goddard Space Flight Center[4]. These data analysis all confirm the existence of anomalous acceleration with the following basic properties: the direction of the anomalous acceleration of the spacecraft towards the Sun, the anomalous acceleration

appears close to $20AU$ and up to $70AU$, the anomalous acceleration seems to be a constant with 10% order of temporal and spatial variations of the anomaly's magnitude. Turyshev *et al.*[5] summarized recent results on researches of the anomaly. Several conventional physical mechanisms have been proposed to explain the anomaly, such as the unknown systematic—the gas leaks from the propulsion system or a recoil force due to the on-board thermal power inventory, and the conventional gravitational force due to a known mass distribution in the outer solar system—the Kuiper Belt Objects or dust, and the expansion of the universe motivated by the numerical coincidence $a_P \simeq cH_0$. However, it was pointed out that these conventional physical mechanisms can not be the answer of the anomalous accelerations[2, 6].

The failure of the conventional physical mechanisms imply that the Pioneer anomaly may correspond to ‘new physics’. One of the most popular ‘new physics’ is the dark matter hypothesis. A specific distribution of dark matter in the solar system would yield the wanted result[7]. However, this special distribution of dark matter is not like the consequence of gravity. Thus, to explain the Pioneer anomaly, dark matter hypothesis still need more work. Several modified gravitational theories also was suggested to explain the Pioneer anomaly, such as the scalar-tensor vector gravity (STVG)theory[8], brane-world models with large extra dimensions[9], and conformal gravity with dynamic mass generation[2]. These modified gravitational theories seem appealing, however, most of them either much more complicated or involves too much hypothesis which does not verified by experiments.

Finsler geometry, which takes Riemann geometry as its special case, is a good candidate to solve the facing problems of the theory of gravitation. The gravity in Finsler space has been studied for a long time[10, 11, 12, 13]. In our previous paper[14], a modified Newton's gravity was obtained as the weak field approximation of the Einstein's equation in Finsler space of Berwald type. We have shown that the prediction of the modified Newton's gravity is in good agreement with the rotation curves of spiral galaxies without invoking dark matter hypothesis.

Randers space, as a special kind of Finsler space, was first proposed by G. Randers[15]. Within the framework of Finsler geometry, modified dispersion relation of free particle in Randers space has been discussed[16]. A modified Friedmann model in Randers space is proposed. It is showed that the accelerated expanding universe is guaranteed by a constrained Randers-Finsler structure without invoking dark energy[17].

In this Letter, in the framework of Finsler geometry we will try to give a simple and clear description of the Pioneer anomaly. As well-known, the length in Riemann geometry is a function of positions. However, this is not the case in Finsler geometry. In Finsler geometry, the length is a function of both position and velocity. Finsler geometry is base on the so called Finsler structure F with the property $F(x, \lambda y) = \lambda F(x, y)$, where x represents position and y represents velocity. The Finsler metric is given as[18]

$$g_{\mu\nu} \equiv \frac{\partial}{\partial y^\mu} \frac{\partial}{\partial y^\nu} \left(\frac{1}{2} F^2 \right). \quad (1)$$

The Randers metric is a Finsler structure F on TM of the form

$$F(x, y) \equiv \alpha(x, y) + \beta(x, y) , \quad (2)$$

where

$$\begin{aligned} \alpha(x, y) &\equiv \sqrt{\tilde{\alpha}_{\mu\nu}(x)y^\mu y^\nu} \\ \beta(x, y) &\equiv \tilde{b}_\mu(x)y^\mu. \end{aligned} \quad (3)$$

Here $\tilde{\alpha}$ is a Riemannian metric on the manifold M . In this Letter, the indices decorated with a tilde are lowered and raised by $\tilde{\alpha}_{\mu\nu}$ and its inverse matrix $\tilde{\alpha}^{\mu\nu}$, otherwise lower and raise the indices are carried by $g_{\mu\nu}$ and $g^{\mu\nu}$. We will show that the Finsler structure in Randers space with \tilde{b} taking the specific form $\tilde{b}_\mu = \{-kr, 0, 0, 0\}$ corresponds to the Pioneer anomaly. The above form of \tilde{b} is given in spherical coordinate and k is a constant.

The parallel transport in Finsler space has been studied in terms of Cartan connection[20, 21, 22]. The notation of parallel transport in Finsler manifold means that the length $F\left(\frac{d\sigma}{d\tau}\right)$ is constant. Following the calculus of variations, one gets the autoparallel equation in Finsler space as[18]

$$\frac{d^2\sigma^\lambda}{d\tau^2} + \gamma^\lambda_{\mu\nu} \frac{d\sigma^\mu}{d\tau} \frac{d\sigma^\nu}{d\tau} = 0. \quad (4)$$

The autoparallel equation (4) is directly derived from the integral length of σ

$$L = \int F\left(\frac{d\sigma}{d\tau}\right) d\tau, \quad (5)$$

the inner product $\left(\sqrt{g_{\mu\nu} \frac{d\sigma^\mu}{d\tau} \frac{d\sigma^\nu}{d\tau}} = F\left(\frac{d\sigma}{d\tau}\right)\right)$ of two parallel transported vectors is preserved. To get a modified Newton's gravity, we consider a particle moving slowly in a

weak stationary gravitational field[19]. Here, we suppose that the Riemannian metric $\tilde{\alpha}$ is close to Minkowskian metric, and $|\tilde{b}_\mu \tilde{b}^\mu|$ is very small

$$\tilde{a}_{\mu\nu}(x) = \tilde{\eta}_{\mu\nu} + \tilde{h}_{\mu\nu}(x), \quad (6)$$

where $\tilde{\eta}_{\mu\nu}$ is the Minkowskian metric and $|\tilde{h}_{\mu\nu}| \ll 1$. Deducing from (4), we obtain the geodesic of Randers space with constant Riemannian speed (namely, $\alpha(\frac{d\sigma}{d\tau})$ is constant)

$$\frac{d^2\sigma^\lambda}{d\tau^2} + \tilde{\gamma}_{\mu\nu}^\lambda \frac{d\sigma^\mu}{d\tau} \frac{d\sigma^\nu}{d\tau} + \tilde{a}^{\lambda\mu} f_{\mu\nu} \alpha \left(\frac{d\sigma}{d\tau} \right) \frac{d\sigma^\nu}{d\tau} = 0, \quad (7)$$

where $f_{\mu\nu} \equiv \frac{\partial \tilde{b}_\mu}{\partial x^\nu} - \frac{\partial \tilde{b}_\nu}{\partial x^\mu}$.

Randers[15] has already found that the Randers metric is related to five dimensional Riemannian geometry. The five dimensional Riemannian metric γ_{mn} ($m, n = 1, 2, 3, 4, 5; \mu, \nu = 1, 2, 3, 4$) is given as

$$\gamma_{\mu\nu} = \tilde{a}_{\mu\nu} - \tilde{b}_\mu \tilde{b}_\nu; \quad \gamma_{\mu 5} = \gamma_{5\mu} = \tilde{b}_\mu; \quad \gamma_{55} = -1. \quad (8)$$

And the geodesic equation (7) of Randers metric is a solution of the five dimensional Einstein's field equation. The five dimensional Einstein tensor is expressed as

$$G^{\mu\nu} = \left(\tilde{R}^{\mu\nu} - \frac{1}{2} \tilde{a}^{\mu\nu} \tilde{R} \right) + \frac{1}{2} \tilde{E}^{\mu\nu}, \quad (9)$$

$$G_5{}^\nu = \frac{1}{2} f^{\nu\mu}{}_{;\mu}, \quad (10)$$

$$G_{55} = \frac{1}{2} \left(\tilde{R} - \frac{3}{4} f^{\mu\nu} f_{\mu\nu} \right), \quad (11)$$

where $\tilde{E}^{\mu\nu} = -f^{\mu\lambda} f_{\lambda}{}^\nu + \frac{1}{4} \tilde{a}^{\mu\nu} f^{\lambda\theta} f_{\lambda\theta}$, $\tilde{R}^{\mu\nu}$ is the Ricci tensor of the four dimensional Riemannian metric α , and the covariant derivative of four dimensional Riemannian metric α is denoted by “;”. In a geometrical viewpoint, the Randers metric arised from

the Zermelo navigation problem [23]. It aims to find the paths of shortest travel time in a Riemannian manifold under the influence of a drift (“wind”). Shen [24] has shown that these minimum time trajectories are exactly the geodesics of a particular Finsler geometry-Randers metric. The map between the Randers metric to a Riemannian space in the viewpoint of Zermelo navigation problem is investigated in the paper [25].

In weak field approximation, the second term of the left side of the equation (7) represents the Newtonian gravitational acceleration. And the third term may induce the anomalous acceleration. One should notice that the trajectory of the Pioneer 10 spacecraft is different from that of the Pioneer 11 spacecraft. The basic property of the Pioneer anomaly that the direction of the anomalous acceleration of the spacecraft towards the Sun tells us that the non vanish components of $f_{\mu\nu}$ is f_{0i} . The term $f_{\mu\nu}$ acts as electromagnetic force. In dealing with the Pioneer anomaly, one need take only the “electric force” into account. Also, due to physical consideration, the “electric force” should be static. Thus, in the approximation of moving slowly and weak field, the geodesic equation (7) reduces to

$$\begin{aligned}\frac{d^2t}{d\tau^2} &= 0, \\ \frac{d^2\sigma^i}{d\tau^2} &= -\frac{1}{2}\frac{\partial h_{00}}{\partial\sigma^i}\left(\frac{dt}{d\tau}\right)^2 - \frac{\partial b_0}{\partial\sigma^i}\alpha\left(\frac{d\sigma}{d\tau}\right)\left(\frac{dt}{d\tau}\right).\end{aligned}\tag{12}$$

The solution of the first equation in (12) is $dt/d\tau = \text{const.}$. Dividing the second equation in (12) by $(dt/d\tau)^2$, we obtain

$$\frac{d\sigma^i}{dt} = -\frac{1}{2}\frac{\partial h_{00}}{\partial\sigma^i} - \frac{\partial b_0}{\partial\sigma^i}.\tag{13}$$

In spherical coordinate, the above equation changes as

$$a = \nabla\varphi + k, \quad (14)$$

where $\varphi \equiv -\frac{GM_\odot}{r}$ is the Newtonian gravitational potential. Then, from the equation (14) one can see clearly that the anomalous acceleration

$$a_p = k. \quad (15)$$

Taking the average value of a_p , we can set the constant k as $9.71 \times 10^{-25} m^{-1}$.

At a position of 20AU far from the Sun, the Newtonian gravitational potential $\varphi = -4.43 \times 10^7 m^2/s^2$ and the perturbation of Minkowskian metric $h_{00} = 9.85 \times 10^{-10}$, and $b_0 = 2.91 \times 10^{-14}$. Thus, the Finsler structure of Randers space is a good description for metric fluctuation around the Minkowskian one. At a position of 1AU far from the Sun, the Newtonian gravitational potential $\varphi = -8.87 \times 10^8 m^2/s^2$ and the perturbation of Minkowskian metric $h_{00} = 1.97 \times 10^{-8}$, and $b_0 = 1.46 \times 10^{-15}$. Einstein's relativity offers high order correction for Newtonian mechanics (post-Newtonian approximation)[19], the corresponding metric correction approximately equals h_{00}^2 . Here, we can see that h_{00}^2 is very close to b_0 . While the Pioneer is not far from the earth, it is hard to distinguish the effect of general relativity(or Riemann geometry) and Finsler geometry. This is a reason for why the anomaly appears in the position of 20AU far from the Sun.

The equation (14) implies that the modified gravitational potential is

$$\varphi_P = krc^2, \quad (16)$$

where c is the speed of light. Since the parameter k is set as $9.71 \times 10^{-25} m^{-1}$, the ration $|\frac{\varphi_P}{\varphi}|$ is less than 10^{-8} for the solar system. The classical tests of general relativity are carried in solar system. Thus, the geodesic equation (7) also predict the same astrophysical phenomena that Einstein's general relativity are able to predict. One also could directly obtain this fact from the field equation (9), for the tensor $\tilde{E}^{\mu\nu}$ in it is the second order in $f^{\mu\nu}$.

The existence of the Pioneer anomaly suggests the Newton's theory of gravitation and general relativity need to be modified even in the solar system. Here, we have suggested that Finsler geometry could give a clear and simply description of the Pioneer anomaly. The specific Finsler structure of the Randers space corresponds to the Pioneer anomaly. We hope that the gravity anomalies mentioned in the beginning of the Letter can be solved systematically in the framework of Finsler geometry.

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